Economics 519 Final Exam

University of Arizona

Fall 2015

Closed-book part: Don't consult books or notes until you've handed in your solutions to Problems #1 - #3. After you do consult books or notes, your solutions to Problems #1 - #3 will not be accepted.

1. Let ϵ be a positive real number and let (X, d) be a metric space in which the metric d satisfies the condition $x' \neq x \Rightarrow d(x, x') \geqq \epsilon$.

(a) Which sequences in X converge and which sequences don't converge? Prove that your answer is correct.

(b) For which sets X is such a metric space (X, d) compact, and for which sets X is such a space not compact? Prove that your answer is correct. ("Compact" here means that the set has the Bolzano-Weierstrass Property.)

2. Let X_1 and X_2 be sets in $\mathbb{R}^n$; let $\widehat{\mathbf{x}}_1 \in X_1$ and $\widehat{\mathbf{x}}_2 \in X_2$; let $\widehat{\mathbf{x}} = \widehat{\mathbf{x}}_1 + \widehat{\mathbf{x}}_2$; and let $\mathbf{p} \neq \mathbf{0} \in \mathbb{R}^n$. Prove that $\widehat{\mathbf{x}}$ maximizes $\mathbf{p} \cdot \mathbf{x}$ on $X_1 + X_2$ if and only $\widehat{\mathbf{x}}_1$ maximizes $\mathbf{p} \cdot \mathbf{x}$ on X_1 and $\widehat{\mathbf{x}}_2$ maximizes $\mathbf{p} \cdot \mathbf{x}$ on X_2.

3. Let S be a convex subset of $\mathbb{R}^n$; let $f_1 : S \to \mathbb{R}$ and $f_2 : S \to \mathbb{R}$ be concave functions; and let $f : S \to \mathbb{R}$ be defined by $\forall x \in S : f(x) = \min\{f_1(x), f_2(x)\}$. Prove that f is concave.

Open-book part: Be sure to turn in your solutions to Problems #1 - #3 before using notes.

4. Provide an example of a sequence $\{f_n\}$ of continuous real functions defined on the unit interval $[0, 1]$ — *i.e.*, functions in $C([0, 1])$ — that converges pointwise to a continuous function f in $C([0, 1])$ but does not converge uniformly — *i.e.*, the sequence does not converge in the normed vector space $C([0, 1])$ with the sup-norm. Prove that indeed your sequence does converge pointwise and does not converge uniformly.

5. Provide a proof by induction that between any two rational numbers there are infinitely many rational numbers — *i.e.*, that if a and b are rational numbers then for every $n \in \mathbb{N}$ there are n rational numbers x that satisfy $a < x < b$.

6. Let (X, d_X) and (Y, d_Y) be metric spaces; let $F_1 : X \twoheadrightarrow Y$ and $F_2 : X \twoheadrightarrow Y$ be correspondences; and let $F : X \twoheadrightarrow Y$ be the correspondence defined by $\forall x \in X : F(x) = F_1(x) \cup F_2(x)$. Prove that if F_1 and F_2 are UHC then F is UHC.